

Hypothesis Testing

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1. Hypotheses and null distributions
 - ▶ Null vs. alternative hypotheses, null distributions via simulation
2. Measuring evidence
 - ▶ p-values, guidelines
3. Common misconceptions
 - ▶ Large p-values, effect size, ignoring study design

An experiment published in *Nature* explored whether infants have preference towards friendly behavior. 16 infants repeatedly watched demonstrations of two scenarios:

- ▶ A “helper” toy assisting the main character
- ▶ A “hinderer” toy blocking the main character

After watching these demonstrations, 14 of 16 infants chose the “helper” toy. The researchers were careful to randomize the color and shape of each character. Do the results of this study suggest that the infants can understand friendly behavior?

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- 3) What is a *model* that could simulate the outcomes we'd expect if $H : p = 0.5$ is true?
 - ▶ Flipping a coin 16 times is akin to “replicating” this experiment under the hypothesis that $p = 0.5$

Statistical tests involve two major components:

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- 2) Deciding whether the sample data provide *sufficient evidence* to falsify the null hypothesis
 - ▶ A **null distribution** describes the outcomes that could have occurred *had the null hypothesis been true*
 - ▶ Evidence against H_0 comes from comparing the outcome observed from the *real data* versus the null distribution

- 1) Using the Randomization test for a single proportion menu of StatKey, input the results of the infant toy choice experiment and verify the null hypothesis $H_0 : p = 0.5$ is being used.
- 2) Click “generate 1000 samples” to create 1000 simulated study outcomes (under the assumption that the null hypothesis is true). How compatible is the observed outcome, $\hat{p} = 14/16 = 0.875$, with these simulated outcomes?

Practice (solution)

- 1) These steps are performed on StatKey
- 2) A proportion of 0.875 or larger happens in less than 1% of simulations, which suggests that such an outcome would be *very unlikely* if the null hypothesis were true. This small probability should be considered *strong evidence* against the null hypothesis.

A **p-value** is the probability of observing an outcome at least as unusual as the one observed in the real data under the assumption that the null hypothesis is true.

- ▶ A p -value is found by looking at either one tail (1-sided test) or both tails (2-sided test) of the null distribution
- ▶ A small p -value can be used to falsify the null hypothesis, but a large p -value should be considered “inconclusive”

P-values as a measure of evidence

Below are the original guidelines put forth by Ronald Fisher (creator of the p -value):

p-value	Evidence against the null
0.100	Borderline
0.050	Moderate
0.025	Substantial
0.010	Strong
0.001	Overwhelming

Fisher intended the p -value to be a quantitative measurement describing the strength of the evidence the sample data provide against a null hypothesis.

Many scientific fields have adopted $\alpha = 0.05$ as a threshold for “statistical significance”

- ▶ Data yielding a p -value smaller than $\alpha = 0.05$ are seen as sufficient evidence for rejecting H_0
- ▶ Data yielding a p -value larger than $\alpha = 0.05$ provide insufficient evidence and result in a “failure to reject H_0 ”

This black and white approach has its flaws, but it's still very widely used.

We've previously discussed a study conducted by Johns Hopkins University that found 31 of 39 babies born 15 weeks early went on to survive. Do these data provide compelling evidence that a majority of babies born 15 weeks early survive?

- 1) Propose a null hypothesis and an alternative hypothesis
- 2) Use StatKey to create a null distribution and find the p -value measuring the evidence this study's observed outcome provides against the null hypothesis
- 3) Use a threshold of $\alpha = 0.05$ to make a decision regarding the null and alternative hypotheses

Practice (solution)

- 1) $H_0 : p = 0.5$, the null hypothesis is that 50% of babies born 15 weeks early survive vs. $H_a : p > 0.5$, the alternative that more than 50% survive
- 2) The 1-sided p -value should be zero (or very close to zero)
- 3) Because the p -value is so small, we reject H_0 in favor of H_a and conclude that these data provide *strong evidence* that more than 50% of babies born 15 weeks prematurely will survive

Probability models instead of simulation

- ▶ Notice how the *simulated null distribution* created by StatKey resembles a Normal distribution
 - ▶ This is not a coincidence, and we can use the following *null model* for a single proportion:

$$\hat{p} \sim N\left(p_0, \sqrt{\frac{p_0*(1-p_0)}{n}}\right)$$

- ▶ p_0 is the *null proportion* (that is specified in the null hypothesis)
- ▶ This model is used to represent outcomes we'd expect if the null hypothesis were true

Wikipedia claims that 70% of babies born 15 weeks early will survive. Do the data in the Johns Hopkins University study (where 31 of 39 babies survived) provide compelling evidence against Wikipedia's claim?

- 1) Propose a null hypothesis and an alternative hypothesis
- 2) Find a Normal model for the null distribution, then use it to determine the p -value
- 3) Use a threshold of $\alpha = 0.05$ to make a decision regarding the null and alternative hypotheses

Practice - part 2 (solution)

- 1) This time, $H_0 : p = 0.7$, and $H_a : p \neq 0.7$ (since either a larger than expected or smaller than expected result would disprove Wikipedia's claim)
- 2) This time, the 2-sided p -value is approximately 0.13
- 3) Because this p -value is larger than 0.05, there's *insufficient evidence* to reject H_0 . It's unclear whether Wikipedia's claim is true, but these data are relatively compatible with it.

Decision errors

In reality, any conclusion drawn from a hypothesis test may or may not be correct:

		The Truth	
		H_0 True	H_0 False
My Decision	Reject H_0	Type I Error	OK
	Fail to Reject H_0	OK	Type II Error

- ▶ A **type I error** occurs when the null hypothesis is *rejected*, but in reality it is *true*
- ▶ A **type II error** occurs when the null hypothesis *cannot be rejected*, but in reality it is *false*

- ▶ The likelihood of a statistical test resulting in a Type I error can be controlled by α , the threshold used for “statistical significance”
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- ▶ Trivially, we could set $\alpha = 0$ and *never* make a Type I error, but how would this impact the likelihood we make a Type II error?
 - ▶ If we make it harder to reject H_0 (ie: we lower α), it becomes easier to make a Type II error

Jury trials in the US use the premise “innocent until proven guilty”. Relating this to hypothesis testing, we can view a trial as a test of H_0 : Person A is not guilty vs. H_a : Person A is guilty

- 1) In words, what would a Type I and Type II error each represent in this scenario?
- 2) Which error would be worse? How might you choose α to be mindful of the trade-off between Type I and Type II errors?

- 1) A Type I error is convicting an innocent person. A Type II error is letting a guilty person go free.
- 2) A Type I error should be viewed as worse, so we might set a very strict decision threshold (ie: $\alpha = 0.01$ or even $\alpha = 0.001$). This is what courts actually do, as the standard of “beyond a reasonable doubt” is generally considered to be a very high bar.

- ▶ Part of the rationale for $\alpha = 0.05$ is that scientific research should always be replicated
 - ▶ Even if one study has a 5% chance of producing a false positive result (Type I error), the chances that three different studies each independently produce a Type I error is $0.05^3 = 0.000125$, or roughly 1 in 10,000
- ▶ Type II errors can be more insidious since findings that aren't statistically significant generally aren't published

Common mistake #1 - interpreting high p -values

As a silly example, suppose Prof. Miller and Steph Curry compete in a 3-point shooting contest. Further, suppose that Prof. Miller makes 3 of 5 and Steph Curry makes 5 of 5.

- ▶ We might use these data to test the hypothesis that Steph Curry and Prof. Miller are equally good 3-pt shooters:
 $H_0 : p_1 - p_2 = 0$
- ▶ The result is a p -value of 0.17, but does that mean that Prof. Miller and Steph Curry are equally good?

Common mistake #1 - interpreting high p -values

- ▶ A high p -value indicates the data provide insufficient evidence against the null hypothesis (not that the null hypothesis is likely true!)
 - ▶ Sample size was an important factor in the Steph Curry example, as 5 shot attempts isn't enough data to make a statistically justified decision

Common mistake #2 - statistical vs. practical significance

- ▶ In the 1980s, AstraZeneca developed *Prilosec*, a highly successful heartburn medication
 - ▶ The FDA patent for Prilosec expired in 2001, prompting AstraZeneca to try to replace Prilosec with a new drug, *Nexium*
- ▶ In a clinical trial comparing the two drugs, Prilosec had a healing rate of 87.5%, while Nexium had a healing rate of 90%

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- ▶ In a clinical trial comparing the two drugs, Prilosec had a healing rate of 87.5%, while Nexium had a healing rate of 90%
 - ▶ The sample size of the trial was very large (over 6000 participants) and the difference between the drugs was “highly significant” with a p -value ≤ 0.01

Common mistake #2 - statistical vs. practical significance

- ▶ p -values depend upon both *sample size* and **effect size**
 - ▶ It's possible for an effect size to be small enough to make *no practical difference*, but for the p -value to be very small (due to a large sample size)
- ▶ Avoid interpreting a very small p -value as being indicative of a very important scientific finding

Common mistake #3 - ignoring study design

- ▶ In the 1970s, UC-Berkeley was investigated for possible sex-discrimination in admissions to its graduate programs
 - ▶ For the fall semester of 1973, 3715 of 8442 male applicants were accepted, but only 1512 of 4321 female applicants were accepted (a difference in proportions of 0.09)
 - ▶ Using StatKey, how does the observed difference in proportions compare to the null distribution corresponding to $H_0 : p_m - p_f = 0$?

Common mistake #3 - ignoring study design

The p -value in the UC-Berkeley example is less than 0.0001, but here's what the data look like when *stratified* by department:

Department	All		Men		Women	
	Applicants	Admitted	Applicants	Admitted	Applicants	Admitted
A	933	64%	825	62%	108	82%
B	585	63%	560	63%	25	68%
C	918	35%	325	37%	593	34%
D	792	34%	417	33%	375	35%
E	584	25%	191	28%	393	24%
F	714	6%	373	6%	341	7%

- ▶ Clearly the overall difference in male - female acceptance can be explained by the *confounding variable* of department
 - ▶ Males tended to disproportionately apply to programs with higher acceptance rates

Proper scientific reporting of a statistical test should include the following:

- 1) An effect size, such as a point estimate and/or a confidence interval
- 2) The p -value itself, not just whether it's above or below 0.05
- 3) A practical conclusion, not just whether to reject H_0 or not reject H_0

Below are 4 different sentences that report the results of the same study. Rank them from best to worst.

- 1) The studied provided compelling evidence to reject the hypothesis that Nexium and Prilosec are equally good.
- 2) The study found that Nexium offered a statistically significant improvement over Prilosec, with a p -value less than 0.01
- 3) The study found that Nexium had significantly higher healing rate than Prilosec (90% vs. 87.5%, $p = 0.003$)
- 4) According to the study, Nexium was found to be significantly better at treating heartburn than Prilosec

Practice (solution)

- ▶ The best is #3, it provides an effect size, the exact p -value, and reasonable summary
- ▶ Next is #2, which provides some indication of the p -value and a reasonable summary
- ▶ Next is #4, which at least gives reasonable summary
- ▶ The worst is #1, it doesn't provide any meaningful insight regarding which treatment should be preferred

Steps of a hypothesis test:

- 1) State the null and alternative hypothesis
- 2) Find the null distribution
- 3) Compare the observed outcome against the null distribution to find the p -value
- 4) Use the p -value (and the effect size) to make a conclusion

Scenarios we've encountered and the proper hypotheses:

- ▶ A single proportion - $H_0 : p = _$ (sometimes 0.5, sometimes depends on the context)
- ▶ A single mean - $H_0 : \mu = _$ (depends on the context)
- ▶ A difference in proportions - $H_0 : p_1 - p_2 = _$ (usually zero)
- ▶ A difference in means - $H_0 : \mu_1 - \mu_2 = _$ (usually zero)

Mistakes to avoid:

- 1) Performing multiple tests in a single experiment without acknowledging the Type I error rate
- 2) Believing that a large p -value means the null hypothesis is likely to be true
- 3) Ignoring effect size when interpreting a p -value (p -values do not measure scientific or practical importance)
- 4) Ignoring study design when interpreting a p -value (confounding variables and biases are not considered in the p -value calculation)