One-Sample Hypothesis Testing

Part 1 - Sampling from a population

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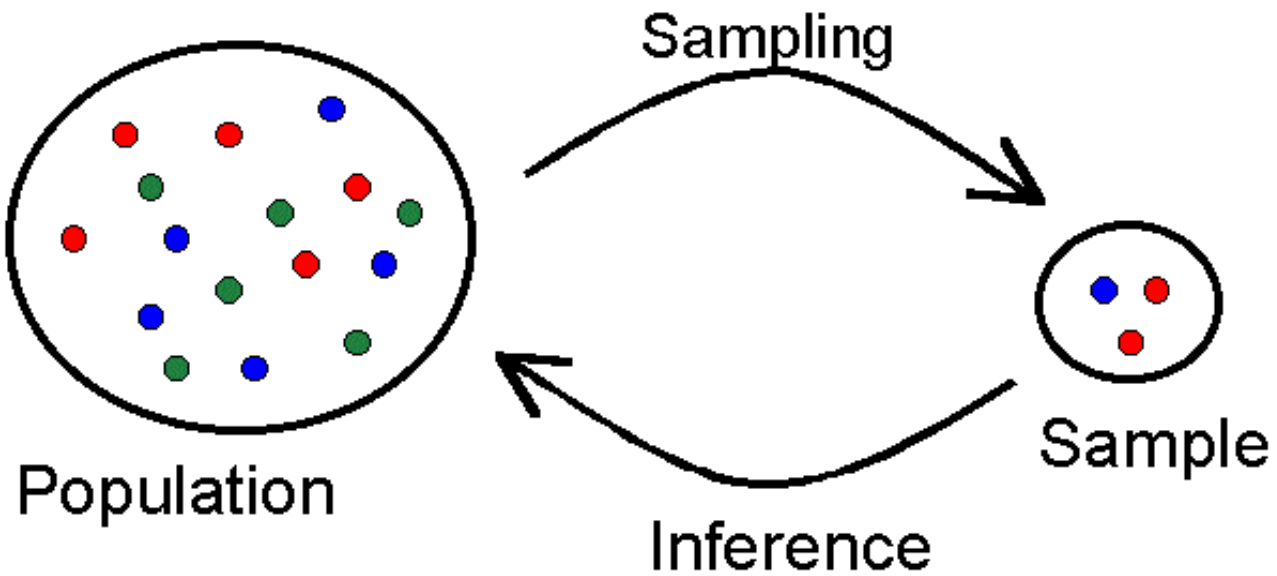
Sampling from a Population

Statisticians adopt the paradigm that the data we are analyzing are a **sample**, or subset of a broader **population**:



In hypothesis testing we make our conjectures and conclusions about the population, using the sample data as evidence

Sampling from a Population

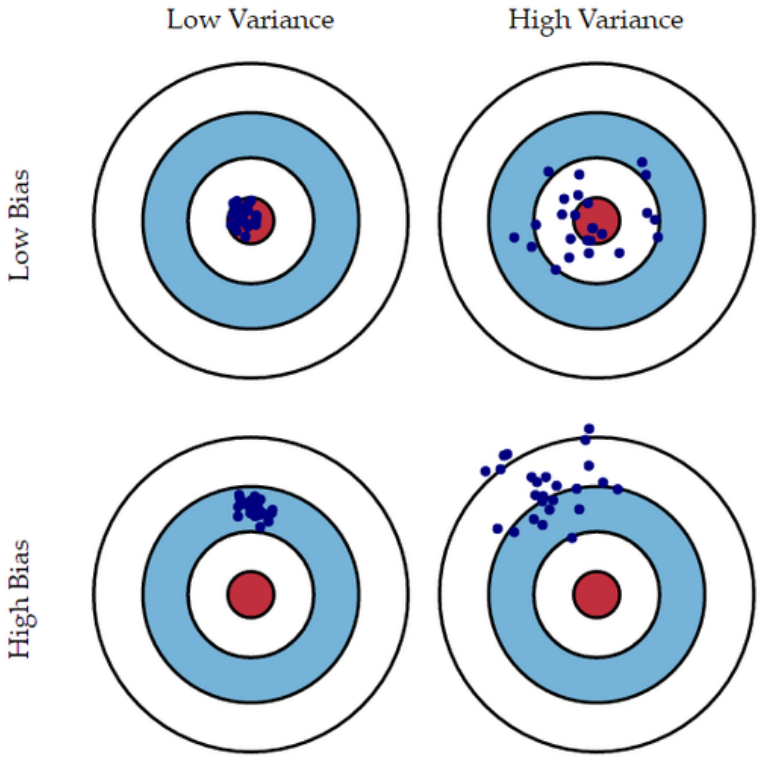
Statisticians use *statistical notation* to distinguish **population parameters** (things we want to know) from **estimates** (things derived from a sample):

	Population Parameter	Estimate (from sample)
Mean	μ	$\overline{x}$
Standard Deviation	σ	s
Proportion	p	$\hat{p}$
Correlation	ρ	r

In our helper-hinderer example, we observed $\hat{p} = 14/16$ and evaluated the population-level hypothesis $p = 0.5$

Sampling Error

How the sample data are collected will influence how accurately they reflect the population they came from:



Note: Here each dot represents an estimate from a *different sample*.

Sources of Sampling Error

There are 2 main reasons for a sample estimate to differ from what's true of the population:

1) **Sampling Bias** - a systematic flaw in the way cases were selected that leads to certain types of cases being disproportionately represented in the sample data
2) **Sampling Variability** - since a sample doesn't include all of the population, any individual sample might differ from the population due to *random chance* (ie: "the luck of the draw")

Having a larger sized sample will *reduce sampling variability* but *will not fix sampling bias*.

Sampling Strategies and Hypothesis Testing

- Sampling bias can be eliminated by taking a **random sample** of cases from the population of interest
  - Other sampling approaches may or may not yield biased samples
    - Surveying students as they leave Noyce for lunch might yield a minimally biased sample of Grinnell science students
    - Surveying students in CSC-301 will yield a biased sample where computer science majors are over-represented

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- The other source of error, sampling variability, can be addressed using **hypothesis testing**!!

Hypothesis Testing

Formalizing the steps from our previous lab, hypothesis testing involves two major components:

1) Proposing a **null hypothesis**, H_0, and an **alternative hypothesis**, H_a
   - The null hypothesis is falsifiable statement about the population that the researchers would like to disprove
   - The alternative hypothesis represents the conclusion the researchers would like to establish

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   - The null hypothesis is falsifiable statement about the population that the researchers would like to disprove
   - The alternative hypothesis represents the conclusion the researchers would like to establish
2) Deciding whether the sample data provide *sufficient evidence* to falsify the null hypothesis
   - A **null distribution** describes the sample outcomes that could have occurred *had the null hypothesis been true*
   - Evidence against H_0 comes from comparing the outcome observed from the *real data* versus the null distribution

P-values

The **p-value** is the *conditional probability* of observing an outcome at least as unusual as the one observed in the real data under the assumption that the null hypothesis is true. Using some loose notion we might think of this as $Pr(\text{Data}|H_0 \text{ is true})$

- The p-value is calculated using either one tail (1-sided test) or both tails (2-sided test) of the *null distribution*
- A small p-value can be used to falsify the null hypothesis, but a large p-value should be considered "inconclusive"

P-values as Evidence

Below are the original guidelines put forth by Ronald Fisher (creator of the *p*-value):

p-value	Evidence against the null
0.100	Borderline
0.050	Moderate
0.025	Substantial
0.010	Strong
0.001	Overwhelming

Fisher intended the *p*-value to be a quantitative measurement describing the strength of the evidence the sample data provide against a null hypothesis.

Decision Thresholds

Despite Fisher's intentions, many scientific fields have adopted $\alpha = 0.05$ as a decision threshold for "statistical significance":

- Data yielding a p-value *smaller* than $\alpha = 0.05$ are seen as *sufficient evidence* for rejecting H_0
- Data yielding a p-value *larger* than $\alpha = 0.05$ are seen as *insufficient evidence* and result in a "failure to reject H_0"

This black and white approach has flaws, but it's still very widely used.

Types of Hypothesis Tests

- A **one-sample test** involves treating all of the available data as a single sample from the same population
  - For example, the $n = 16$ infants in our helper-hinderer example were analyzed as a single sample intended to represent the behaviors of "all infants"

Types of Hypothesis Tests

- A **one-sample test** involves treating all of the available data as a single sample from the same population
  - For example, the $n = 16$ infants in our helper-hinderer example were analyzed as a single sample intended to represent the behaviors of "all infants"
- In contrast, a **two-sample test** involves the comparison of two different groups, each of which is taken to represent a different population
  - For example, a drug trial might split participants into treatment and control groups and compare the outcomes of each group
  - The treatment group is taken to represent all users of the drug, and the controls are taken to represent non-users

Types of Hypothesis Tests

- The distinction between one-sample and two-sample tests is important because it will guide the hypotheses we set up and the procedure used to determine the null distribution
  - For *one-sample categorical data*, our null hypothesis will have the form $H_0 : p = \_\_\_$
  - For *one-sample quantitative data*, our null hypothesis will have form $H_0 : \mu = \_\_\_$ (we won't test anything other than the mean, at least for now)

Null Distributions for One-Sample Tests

For now we'll create the *null distribution* for our hypothesis tests using simulations:

- For *one-sample categorical data*, we can simulate proportions that could have occurred in sample had H_0 been true
  - You might imagine spinning a spinner with the hypothesized probability of the outcome of interest for each case in the sample
- For *one-sample quantitative data*, we can simulate means that could have occurred in sample had H_0 been true by subtracting the hypothesized mean from each data-point then sampling the re-centered data with replacement

Practice

Researchers at Johns Hopkins University studying preterm birth in the United States recruited consenting volunteers who had a child born at 25-weeks (15 weeks prematurely) in their hospital system. They found that 31 of 39 babies survived.

- **Part A** - What are the *sample* and the *population of interest* in this study? How would you describe the representativeness of this sample?
- **Part B** - The Wikipedia article on preterm birth lists the survival rate of babies born at 25-weeks at 70%. Do the data from this study provide compelling evidence against this claim? State the null and alternative hypotheses for a two-sided test using statistical notation

Practice (cont.)

- **Part C** - What is the *sample estimate* of the parameter described in the null Hypothesis? Report the estimate using proper statistical notation.
- **Part D** - Use the sample data and hypotheses from earlier to find the two-sided p-value. Do these data provide sufficient evidence to refute Wikipedia's claim? Use StatKey to help you estimate the p-value.
- **Part E** - Suppose we had cheated and set up a one-sided alternative hypothesis of $H_a : p > 0.7$ after noticing the data showed a survival rate above 70%. What would the one-sided p-value be in this scenario?

Wrap-up

Most of our analyses in this class will involve some type of hypothesis test. When reporting the results of a test you are expected to include the following:

1. *Scientific context* - Example: the survival of infants born at 25-weeks
2. *Strength of evidence* - Examples: moderate/strong evidence (typically p-value < 0.05), or a lack of evidence (typically p-value ≥ 0.05)
3. *The type of relationship* - Example: higher survival rate

You should *not* use generic phrases like "reject H_0" or "conclude the alternative due to $p < 0.05$" unless explicitly asked to do so.