Practice Exam #3 (Sta-209, S24)

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The following information will appear verbatim on the first page of Exam 3.

Directions

- Answer each question using *no more than specified number of sentences* and not attempt to avoid these guidelines by using run-on sentences. Answers that are unnecessarily verbose may result in point loss.
- Do not include superfluous information in your answers, you may be penalized if you make an inaccurate statement even if you go on to provide a correct answer.

Formula Sheet

Definitions:

- **Risk**: relative frequency of an event/outcome
- **Relative Risk**: ratio of the risks across two groups
- **Odds**: ratio of how often an event/outcome is observed relative to how often it is not observed
- **Odds ratio**: ratio of odds across two groups

Formulas:

$$\bar{x} = \tfrac{1}{n}\sum_{i=1}^{n} x_i$$

$$s = \sqrt{\tfrac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$r = \tfrac{1}{n-1}\sum_{i=1}^{n}\big(\tfrac{x_i-\bar{x}}{s_x}\big)\big(\tfrac{y_i-\bar{y}}{s_y}\big)$$

Statistic	Standard Error	Conditions
$\hat{p}$	$\sqrt{\frac{p(1-p)}{n}}$	$np \geq 10$ and $n(1-p) \geq 10$
$\bar{x}$	$\frac{\sigma}{\sqrt{n}}$	normal population or $n \geq 30$
$\hat{p}_1 - \hat{p}_2$	$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$	$n_i p_i \geq 10$ and $n_i(1-p_i) \geq 10$ for $i \in \{1,2\}$
$\bar{x}_1 - \bar{x}_2$	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	normal populations or $n_1 \geq 30$ and $n_2 \geq 30$

Chi-squared test statistic: $X^2 = \sum_{j=1}^{k} \frac{(\text{observed}_j - \text{expected}_j)^2}{\text{expected}_j}$

Question #1 (conceptual questions)

Part A: Suppose we are interested in building a linear regression model that predicts daily ozone concentration based upon three quantitative explanatory variables: temperature, wind speed, and solar radiation. Identify which of the following statements must be **true** (there may be more than 1 true statement):

A) The model: $\widehat{Ozone} = b_0 + b_1Temp + b_2Wind$ will have a smaller sum of squared residuals than the model $\widehat{Ozone} = b_0 + b_1Solar$

B) The model: $\widehat{Ozone} = b_0 + b_1Temp + b_2Wind$ will have a smaller sum of squared residuals than the model $\widehat{Ozone} = b_0 + b_1Temp$

C) The model: $\widehat{Ozone} = b_0 + b_1Temp + b_2Temp^2$ will have a smaller sum of squared residuals than the model $\widehat{Ozone} = b_0 + b_1Wind$

State which statements are true and briefly explain the reasoning or thought process you used to determine whether a statement was true or false.

- Only statement B is true. In the other scenarios the two models are not nested, so its impossible to tell which model will better fit the data. However, in B the smaller model is nested within the larger one, so we know that adding Wind as a predictor can only improve the sum of squared residuals.

Part B: For each of the following scenarios state the name of the appropriate hypothesis test. You do not need to explain your answers.

- i: Using a sample Grinnell students from the science division to see if the racial/ethnic distribution of science students at Grinnell differs from the distribution of the entire student body that is published by the college.
  - Chi-squared goodness of fit test
- ii: Conducting a randomized experiment to determine if fertilizer A produces a higher average crop yield than fertilizer B.
  - Two-sample T-test (for a difference in means)
- iii: Using a poll that asks $n = 200$ voters if they will vote for candidate A or candidate B to see if there is evidence that candidate A will receive a majority of votes in the election.
  - One-sample Z-test (one proportion)
- iv: Conducting an experiment where 4 different brands of feed supplements are given to piglets with the intent of determining whether there is an association between type of feed supplement during youth and the adult weight of a pig.
  - One-way ANOVA

Part C: Recall that one-way ANOVA can be described as a comparison between two models using the observed sample data. With this in mind, answer the following questions:

- i: Suppose we are interested in how each model involved in one-way ANOVA will predict the value of the outcome variable for a new observation. Briefly describe what the prediction will be based upon for each model.
  - For the Null model the prediction is the overall mean of the entire sample. For the alternative model the prediction depends upon the group that the new data-point belongs to, and it will be that group's mean.
- ii: The models in one-way ANOVA involve the Normal distribution. Briefly describe the role of the Normal distribution in these models.
  - The Normal distribution is the model for the errors (deviations from the mean/prediction) within each model. This model underlies the validity of the F-test that compares these two models.
- iii: Suppose we perform one-way ANOVA and reject the null hypothesis. We check the model's assumptions and they are verified as reasonable. Is this the end of our analysis or is there more that we

should do? If this is end, briefly describe what we'd conclude from the test (in generic terms). If more should be done, briefly describe what you'd do next.

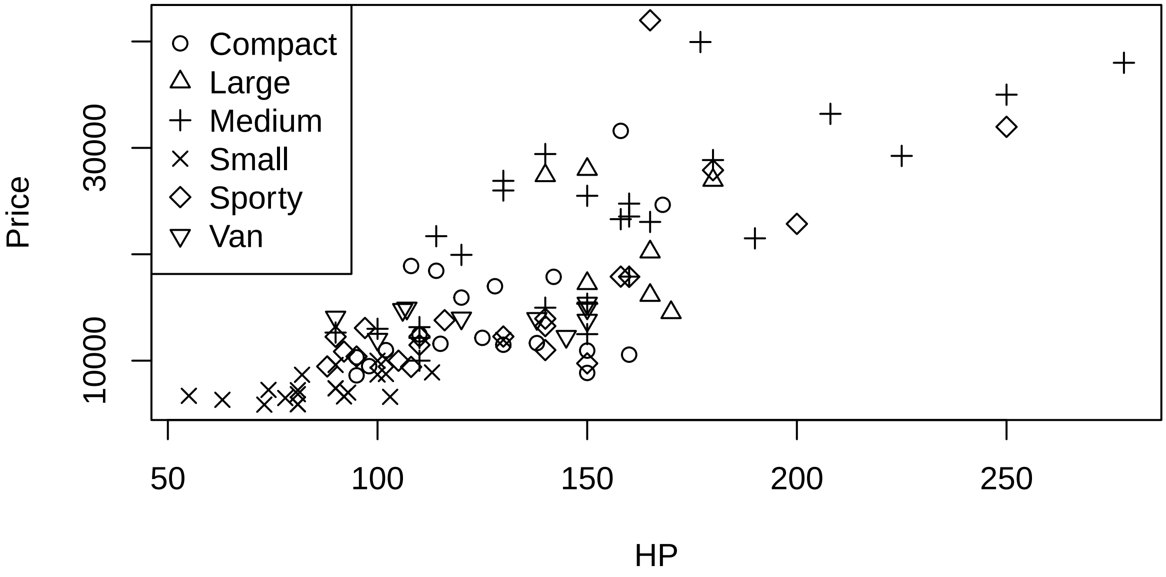
  - After a significant one-way ANOVA we should perform post-hoc testing to explore pairwise differences between groups.

Question #2

This question will analyze data on 111 different types of cars published in *Consumer Reports*. The overall goal of the analysis is to identify factors associated with price. A few key variables include:

- **Price** - List price (US dollars) with standard equipment
- **Country** - Where the car was manufactured
- **HP** - Net horsepower
- **Type** - A categorical variable describing the general type of vehicle (small, medium, large, compact, sporty, van)
- **Length** - Length of the vehicle (inches)

Part A: The plot below depicts the relationship between Price, HP, and Type. Based upon this plot, is HP associated with Price? Is Type associated with Price? Provide a brief explanation of your answers.



- Type appears associated with price, as an example the small cars (shown as X's) tend to have lower prices.
- HP is associated with price, there is a moderate-to-strong, positive linear relationship shown in the graph.

Part B: Ignoring all other variables, what *statistical approach* would be the most appropriate *hypothesis test* for discovering a possible association between Type and Price? Provide the name of the test and a brief explanation (no more than 1-sentence).

- One-way ANOVA

Part C: The table below summarizes price by vehicle type. Is any information presented in this table problematic for the validity of the statistical test you identified in Part A? If so, briefly explain what aspect(s) of these data are problematic.

Type	N	Mean	Median	StdDev
Compact	19	14395.368	11650.0	5938.762
Large	7	21499.714	20225.0	5825.878
Medium	26	22750.154	23170.0	8416.809
Small	22	7736.591	7239.5	1627.928
Sporty	21	15889.810	12279.0	8539.240
Van	10	14014.300	14037.5	1126.104

- Yes, ANOVA assumes equal variability within each group, but some types like "Sporty" have several times the standard deviation in price as types like "Van".

Part D: The table below displays the *coefficient estimates* of a linear regression model that uses both Type and HP to predict a vehicle's Price. Use the information in this table to answer the following questions (I - III)

	Coefficient	Std. Error	t statistic	p-value
(Intercept)	-1842.01	2197.32	-0.84	0.40
HP	128.23	14.91	8.60	0.00
TypeLarge	2825.67	2224.20	1.27	0.21
TypeMedium	4593.94	1543.07	2.98	0.00
TypeSmall	-1810.14	1635.77	-1.11	0.27
TypeSporty	488.56	1556.85	0.31	0.75
TypeVan	-248.79	1915.62	-0.13	0.90

I) The intercept of this model is -1842.01, what does this value mean? Should we care that this value isn't statistically significant?

- This is the expected price of a compact car w/ zero HP. We shouldn't care about this value because no cars have horsepowers of zero.

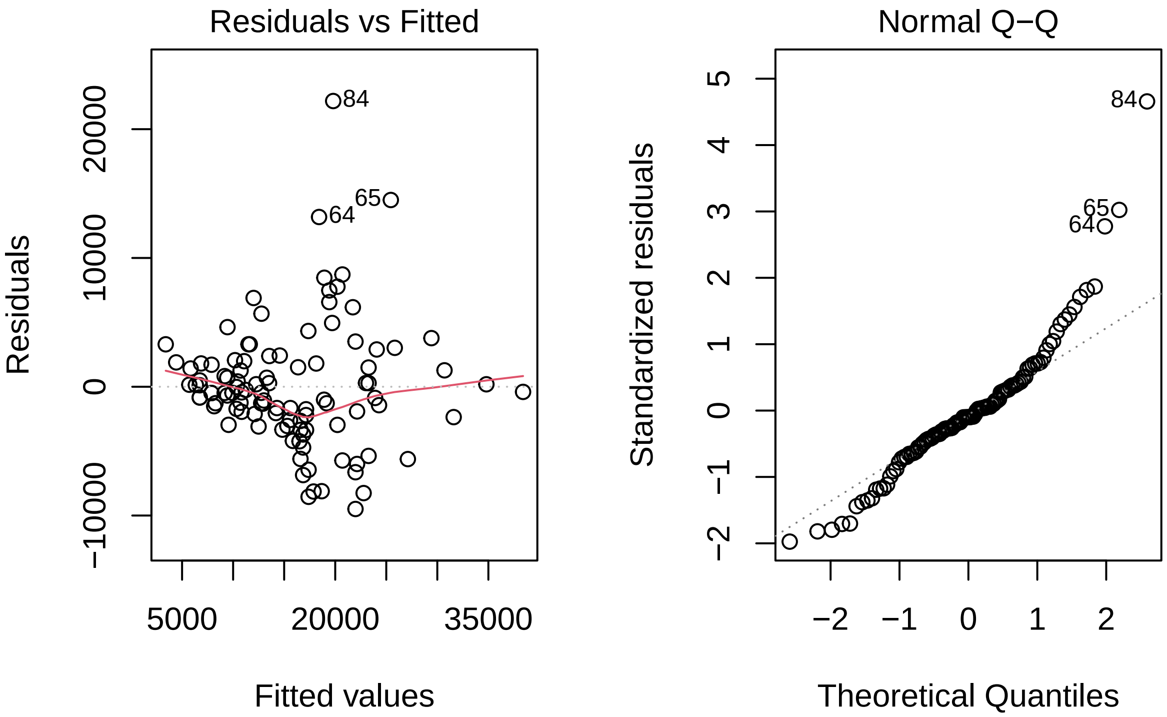
II) Provide a one sentence interpretation of the coefficient for "TypeMedium", be specific.

- Medium cars are expected to sell for $4593 than compact cars with the same horsepower. This is a statistically significant difference and we can be confident that it is larger than what could reasonably be attributed to sampling variability.

III) True or False, in this model the effect of HP on price differs depending on the type of vehicle. You do not need to explain your answer.

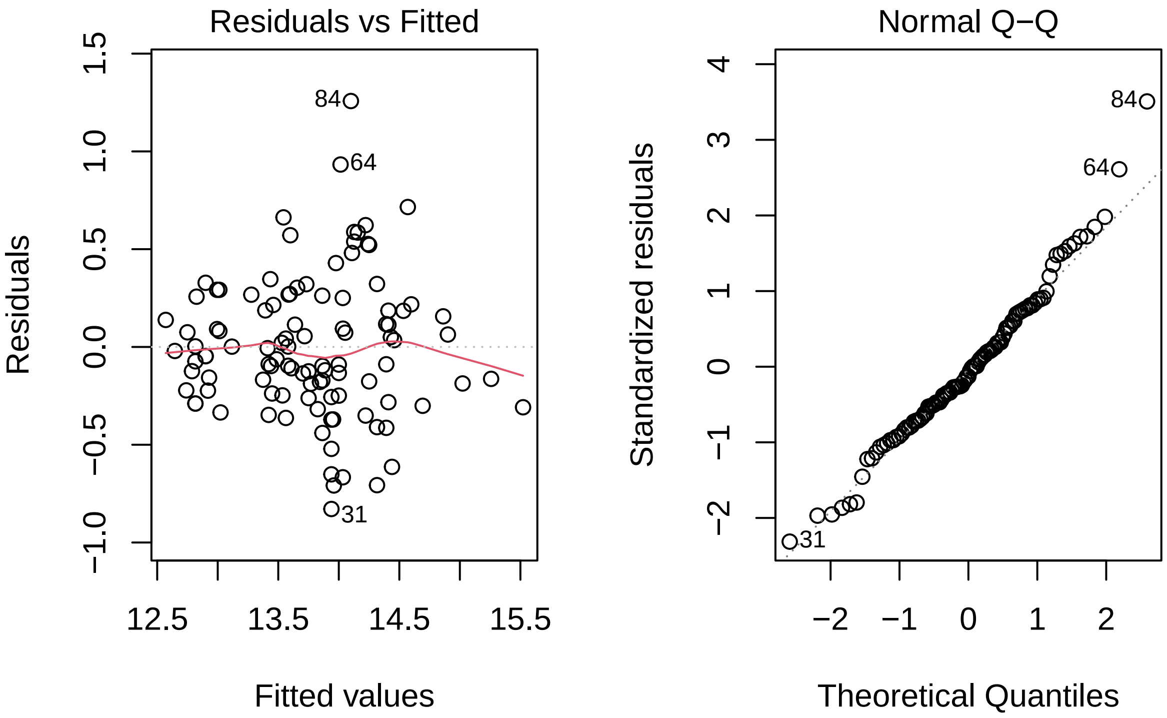
- False

Part E: Below are two `R` plots related to the model described in Part D, `Price ~ HP + Type`. Based upon what you see in these plots, do you believe *p*-values calculated for these data will be valid/reliable? Briefly explain.



- The residuals don't appear to follow a Normal distribution (upper right of the QQ-plot). Though the deviation isn't too horrible, you may want to interpret borderline p-values with caution.

Part F: The plots show results after transforming the response variable `Price` using a log-transformation, making the model: `log2(Price) ~ HP + Type`. When compared with the model from Parts D-E, are you more comfortable trusting the p-values produced by statistical tests that use this model? Briefly explain why or why not.



- Yes, the residuals now follow a Normal distribution almost perfectly. The p-values should be more trustworthy because the assumptions of this model are met.

Part G: Below are statistical results found using R. Based upon what is given, state the null hypothesis of the test that was performed in words and provide a one-sentence conclusion describing the results of the test in regard to the null hypothesis.

```
mod0 <- lm(log2(Price) ~ HP, data = car90)
mod1 <- lm(log2(Price) ~ HP + Type, data = car90)
anova(mod0, mod1)
```

```
## Analysis of Variance Table
##
## Model 1: log2(Price) ~ HP
## Model 2: log2(Price) ~ HP + Type
##   Res.Df    RSS Df Sum of Sq      F    Pr(>F)
## 1    103 19.333
## 2     98 13.347  5    5.9861 8.7906 6.427e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- The null hypothesis is that both models fit the data equally well. We reject this null hypothesis and conclude that including the predictor “Type” significantly improves the fit of the model (ie: it leads to predictions that are closer to the actual prices).

Question #3

In the mid-1860s, Joseph Lister, a Professor of Surgery at the Glasgow Royal Infirmary, conducted an experiment to investigate his hypothesis that harmful micro-organisms were the cause of deadly infections that frequently occurred after surgery. Lister randomly assigned 75 surgical patients to receive either his newly developed "sterile" surgery protocol, which entailed wearing clean gloves, gowns, and disinfecting surgical instruments, or a "control" surgery protocol, where no sterilizations steps were taken prior to surgery.

The results of Lister's experiment are summarized below:

	Died	Survived
Control	16	19
Sterile	6	34

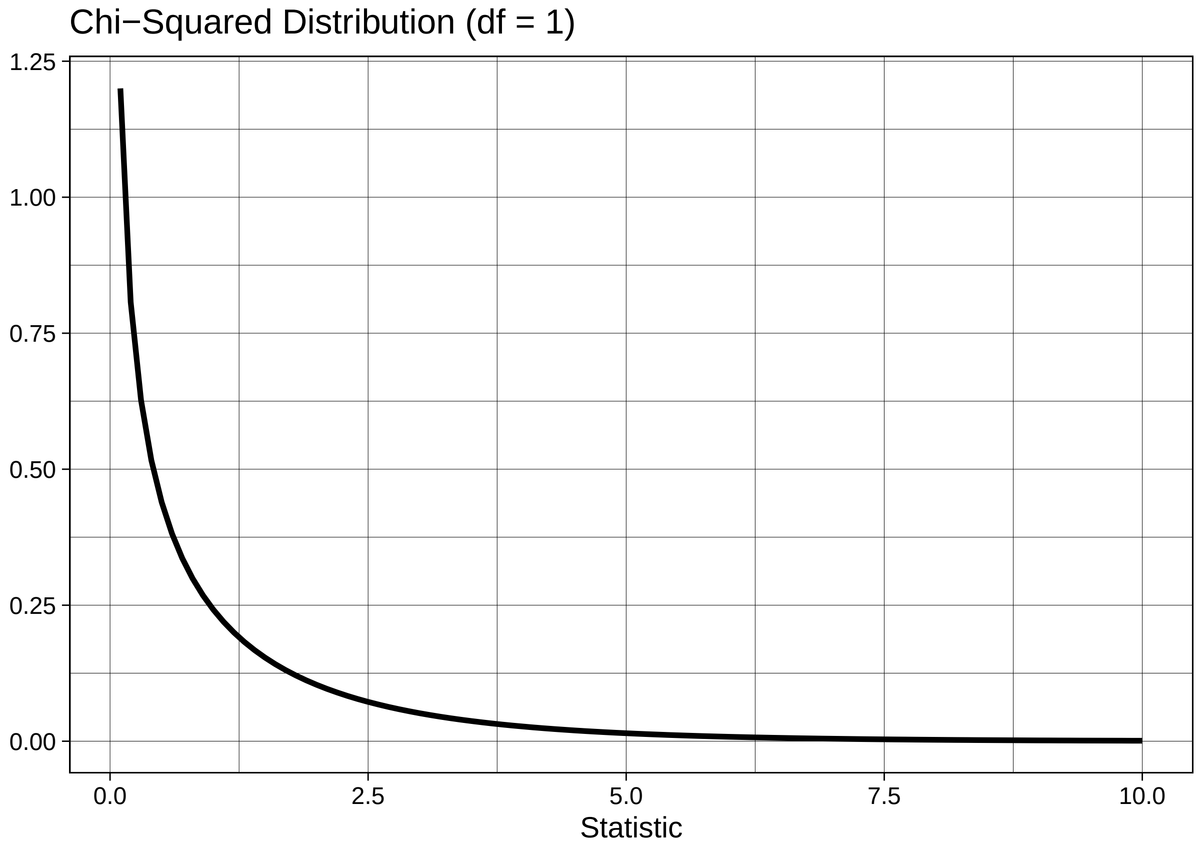
Part A: If Lister's sterilization protocol made no difference, how many deaths and survivals would you expect in each group (sterile and control)? Provide a table of expected counts.

	Died	Survived
Control	10.26667	24.73333
Sterile	11.73333	28.26667

Part B: Use the table of expected counts you found in Part A to calculate a test statistic for a Chi-squared test of association.

- $X^2 = (16-10.267)^2/10.267+(19-24.733)^2/24.733+(6-11.733)^2/11.733+(34-28.267)^2/28.267 = 8.495$

Part C: Shown below is Chi-squared distribution with 1 degree of freedom. Using your results from Part A/B, shade the region of this curve corresponding to the p-value.



- Only the extreme right portion should be shaded (values of the test stat larger than 8.495)

Part D: Estimate the p-value using your response to Part C, and provide a brief conclusion that is consistent with this p-value that involves the context of this hypothesis test.

- The p-value associated with this X^2 value is small, any estimate less than 0.5 is reasonable, noting that the actual p-value is 0.0036. Based upon this we reject the null hypothesis of no association and conclude that Lister's sterilization protocol is associated with increased survival after surgery.