Practice Exam #2 (Sta-209, S25)

Ryan Miller

Formula Sheet

Definitions:

- Type I error - rejection of a true null hypothesis
- Type II error - failure to reject a false null hypothesis

Central Limit theorem results:

Statistic	Standard Error	Conditions
$\hat{p}$	$\sqrt{\frac{p(1-p)}{n}}$	$np \geq 10$ and $n(1-p) \geq 10$
$\bar{x}$	$\frac{\sigma}{\sqrt{n}}$	normal population or $n \geq 30$
$\hat{p}_1 - \hat{p}_2$	$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$	$n_i p_i \geq 10$ and $n_i(1-p_i) \geq 10$ for $i \in \{1,2\}$
$\bar{x}_1 - \bar{x}_2$	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	normal populations or $n_1 \geq 30$ and $n_2 \geq 30$
r	$\sqrt{\frac{1-\rho^2}{n-2}}$	normal populations or $n > 30$

Question #1 (conceptual questions)

Part A: In your own words, explain the meaning of "95% confidence" in the statistical term "95% confidence interval". You should avoid using the words "confidence" or "confident" as core components of your explanation and you should limit yourself to around 2-sentences.

Part B: Suppose your friend is planning to perform a hypothesis test and is considering using a decision threshold of $\alpha = 0.01$ rather the conventional threshold of $\alpha = 0.05$. How will their proposed threshold influence the likelihood of a Type I and a Type II error relative the likelihood of these errors using the conventional threshold.

Part C: In your own words, briefly explain the difference between a "sample" and a "population" in the context of statistical inference. That is, how to the concepts of a "sample" and a "population" relate to statistical procedures like confidence interval estimation and hypothesis testing. You should limit your answer to around 3-sentences.

Part D: Suppose you and your friend are each planning to collect a sample of Grinnell students to estimate the proportion of students who have taken Sta-209. You each plan to present a confidence interval estimate of this proportion based upon the data you collect in your sample. Before collecting any data, and assuming *everything else is the same*, which of the changes would lead your confidence interval estimate to be more likely to contain the true proportion than your friend's? You may simple circle or mark the options that will make your interval more likely to contain the truth.

i) You use a 95% confidence level when your friend only uses a 90% confidence level
ii) You use a procedure that is free of sampling bias, while your friend uses a biased sampling protocol
iii) You sample 50 Grinnell students when your friend only samples 30 Grinnell students

Part E: Suppose a researcher is planning to perform several thousand hypothesis tests on data from an experiment involving a large number of genes. The researcher is primarily concerned with identifying as many important genes as possible, but they also want to have some control over the number of false positive findings. Would you recommend the researcher use a *false discovery rate control* procedure or the *Bonferroni correction*? State your answer and provide a brief (1-2 sentence) explanation

Question #2

An experiment conducted at the University of Sydney in Australia investigated whether electrical stimulation to the brain could help participants successfully solve problems that required non-routine approaches.

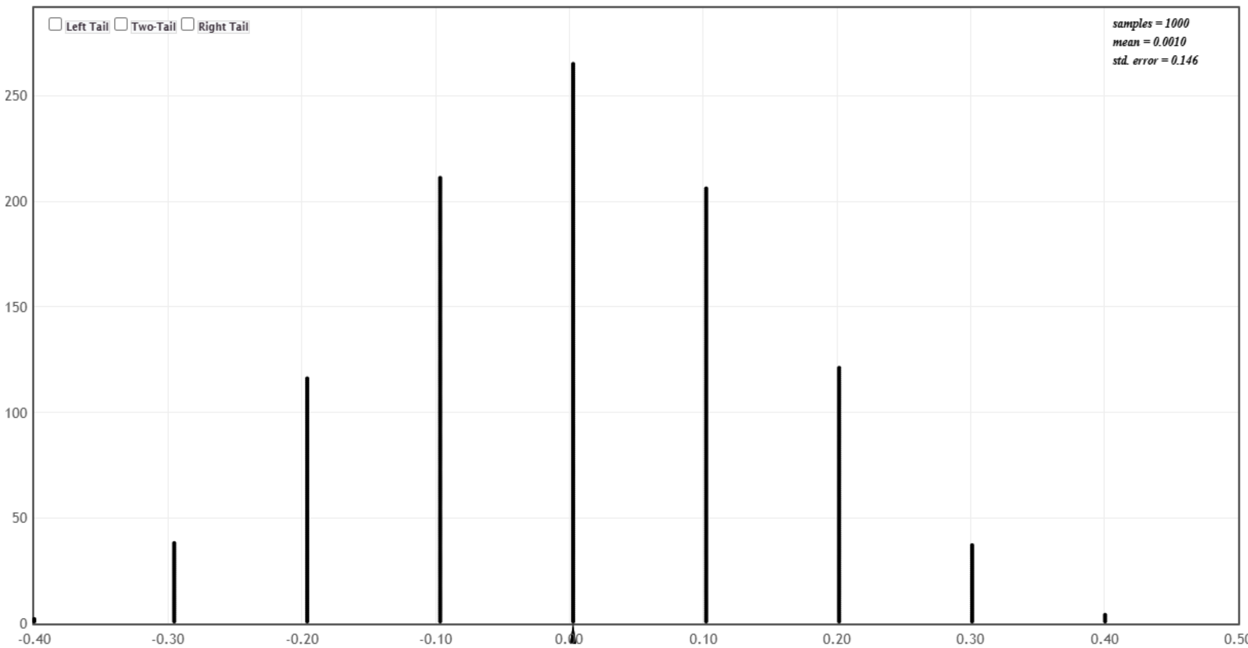
The experiment trained 40 participants to solve problems in a particular way and then asked them solve an unfamiliar problem that required a creative solution. Of the study's participants, 20 were randomly assigned to receive electrical stimulation to the brain prior to working on the unfamiliar problem, and the other 20 received a placebo condition (the same apparatus without any electricity).

In the electrical stimulation group, 8 participants (40%) successfully solved the problem, while only 4 participants (20%) in the placebo group solved the problem.

Part A: Notice that a larger proportion of the electrical stimulation group correctly solved the unfamiliar problem. With this in mind, why is a statistical test necessary for the researchers to conclude that electrical stimulation increases the ability of individuals to solve unfamiliar problems? You should limit your explanation to at most 2-sentences.

Part B: Using *both* words and statistical symbols, state the hypotheses that these researchers should be interested in evaluating.

Part C: The figure below shows a simulated null distribution created using StatKey. Using this distribution, provide an estimate of the two-sided p-value. Your estimate doesn't need to be exact, but it should be reasonably close. You may annotate the figure or show your work to help me assess whether you're estimating your p-value in the proper manner.



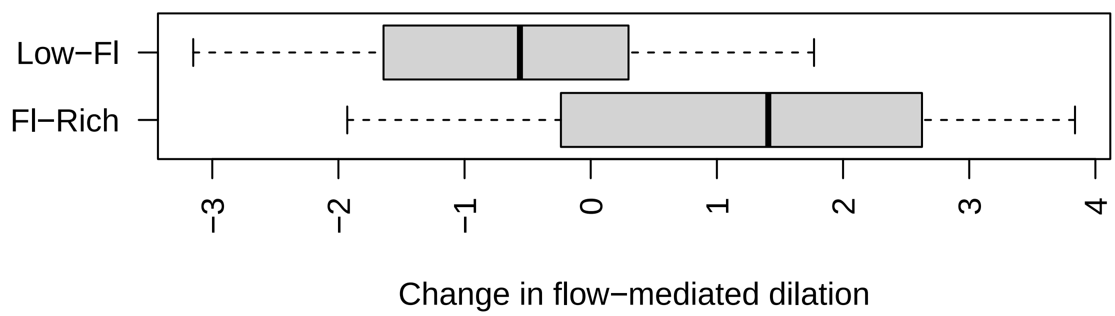
Part D: Using the p-value you estimated in Part C. Provide a 1-2 sentence conclusion that summarizes the results of this study.

Part E: Suppose these data were used to form a 95% confidence interval estimate for the difference in the proportion of individuals who solve unfamiliar problems with and without electrical stimulation. Would you expect your confidence interval to suggest that a difference of zero is plausible? You should limit your explanation to at most 2-sentences.

Question #3

In a 2004 study, 21 participants were randomly assigned to consume either 46g (1.6 oz) of flavonoid-rich dark chocolate or 46g of chocolate with low flavonoid content every day for 2 weeks. Both chocolates are indistinguishable in taste. Participants had their flow-mediated dilation measured before and after the experiment. Larger increases in flow-mediated dilation indicate a greater improvement in vascular health.

For the 11 participants randomized to consume flavonoid-rich chocolate, the mean increase in flow-mediated dilation was 1.3 units (standard deviation of 2.3). For the 10 participants randomized to consume low-flavonoid chocolate, flow-mediated dilation decreased by an average of -0.9 units (standard deviation of 1.5). The figure below summarizes the observed changes in each group:



Part A: Recall that 95% of the Standard Normal distribution is within 1.96 units of the distribution's center. Additionally, recall that Central Limit theorem suggests the standard error of a single mean can be estimated by $s/\sqrt{n}$. Assuming that a Normal model is appropriate, calculate a 95% confidence interval for the mean change in flow-mediated dilation for individuals who consume flavonoid-rich chocolate.

Part B: Based upon the interval you calculated in Part A, can you confidently conclude that the consumption of flavonoid-rich chocolate, on average, increases flow-mediated dilation? Briefly explain.

Part C: Your friend looks at the procedure you used to calculate your confidence interval in Part A and suggests you should use the t-distribution rather than the Standard Normal distribution to calibrate the interval's margin of error. If you implemented your friend's suggestion, would the resulting interval be *wider* or *narrower* than the one you previously had calculated? State your answer and provide a brief (1-2 sentence) explanation.

Part D: Another friend of yours looks at the procedure being considered in Part C (using the t-distribution) and states that the t-distribution is inappropriate for this application be the sample size, $n = 11$, is too small. Do you agree with this friend? State your answer and provide a brief (1-2 sentence) explanation.

Part E: This study included a comparison group in addition to the group receiving high-flavanoid chocolate. Suppose we'd like to perform a statistical test to evaluate whether these two groups experienced different changes in flow-mediated dilation. State the appropriate hypotheses for this test using *both* words and statistical symbols.

Part F: The p-value for the hypothesis test described in Part E is 0.0204. In your own words, explain what this means.