Practice Exam #3 (Sta-209, S25)

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The following information will appear verbatim on the first page of Exam 3.

Directions

- Answer each question using *no more than specified number of sentences* and not attempt to avoid these guidelines by using run-on sentences. Answers that are unnecessarily verbose may result in point loss.
- Do not include superfluous information in your answers, you may be penalized if you make an inaccurate statement even if you go on to provide a correct answer.

Formula Sheet

Theoretical models:

- Linear regression: $y_i = \beta_0 + \beta_1 \cdot x_{i1} + \ldots + \beta_p \cdot x_{ip} + \epsilon_i$
- Logistic regression $\log\left(\frac{Pr(y=1)}{1-Pr(y=1)}\right) = \beta_0 + \beta_1 \cdot x_{i1} + \ldots + \beta_p \cdot x_{ip}$

Chi-squared test statistic:

$$X^2 = \sum_{j=1}^{k} \frac{(\text{observed}_j - \text{expected}_j)^2}{\text{expected}_j}$$

F-test statistic:

$$F = \frac{(SS_0 - SS_1)/(d_1 - d_0)}{SS_1/(n - d_1)}$$

Reminders:

- Odds - a ratio of how often an event occurs relative to how often it doesn't occur
- Linear regression assumptions - residuals are independent and Normally distributed with constant variance

Question #1 (conceptual questions)

Part A: Suppose we are interested in building a linear regression model that predicts daily ozone concentration based upon three quantitative explanatory variables: temperature, wind speed, and solar radiation. Identify which of the following statements must be **true** (there may be more than 1 true statement):

A) The model: $\widehat{Ozone} = b_0 + b_1 Temp + b_2 Wind$ will have a smaller sum of squared residuals than the model $\widehat{Ozone} = b_0 + b_1 Solar$
B) The model: $\widehat{Ozone} = b_0 + b_1 Temp + b_2 Wind$ will have a smaller sum of squared residuals than the model $\widehat{Ozone} = b_0 + b_1 Temp$
C) The model: $\widehat{Ozone} = b_0 + b_1 Temp + b_2 Temp^2$ will have a smaller sum of squared residuals than the model $\widehat{Ozone} = b_0 + b_1 Wind$

State which statements are true and briefly explain the reasoning or thought process you used to determine whether a statement was true or false.

Part B: For each of the following scenarios state the name of the appropriate statistical model/hypothesis test. You do not need to explain your answers.

- i: Using a sample Grinnell students from the science division to see if the racial/ethnic distribution of science students at Grinnell differs from the distribution of the entire student body that is published by the college.
- ii: Conducting a randomized experiment to determine whether fertilizer A, fertilizer B, or fertilizer C will have different crop yields.
- iii: Understanding how highschool GPA, ACT score, and number of extracurricular activities relate to whether or not students are accepted into Ivy League schools.
- iv: Evaluating whether homes are more likely to have solar panels in certain regions of the country than others.

Part C: Recall that one-way ANOVA can be described as a comparison between two models using the observed sample data. With this in mind, answer the following questions:

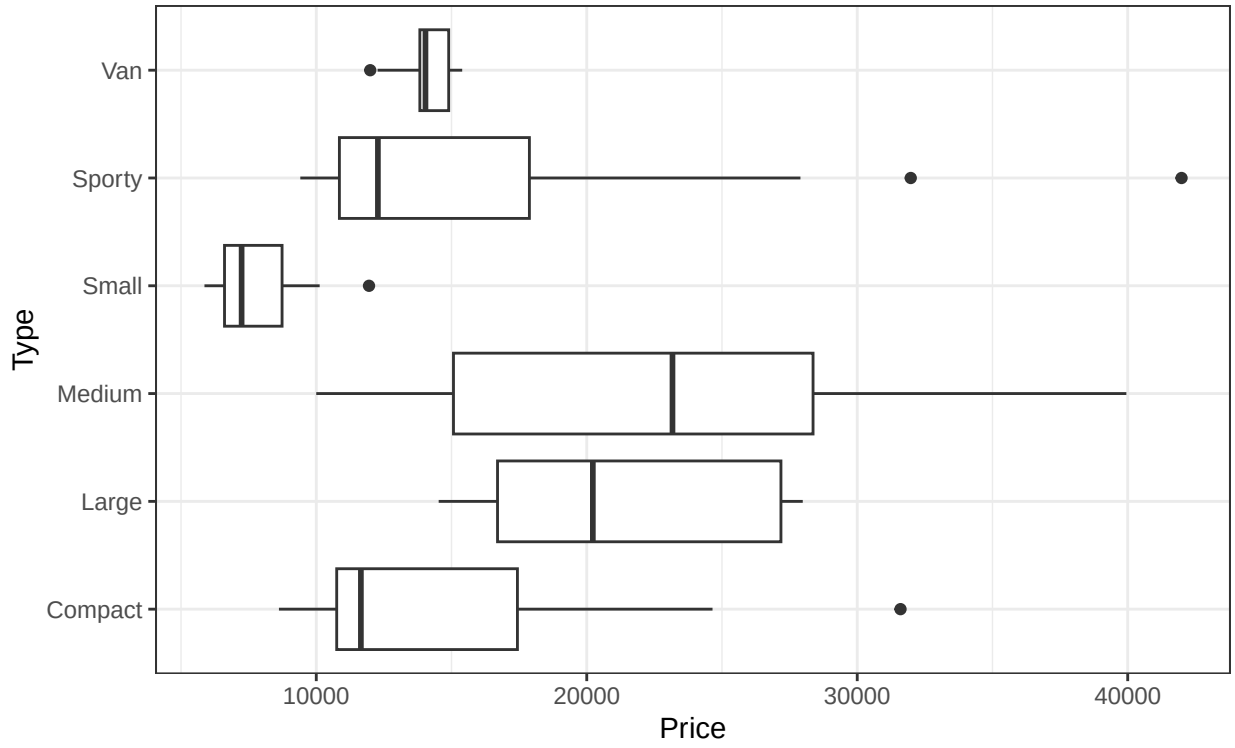
- i: Suppose we are interested in how each model involved in one-way ANOVA will *predict* the value of the outcome variable. Briefly describe what the prediction will be based upon for each model.
- ii: The models in one-way ANOVA involve the Normal distribution. Briefly describe the role of the Normal distribution in these models.
- iii: Suppose we perform one-way ANOVA and reject the null hypothesis. We check the model's assumptions and they are verified as reasonable. Is this the end of our analysis or is there more that we should do? If this is end, briefly describe what we'd conclude from the test (in generic terms). If more should be done, briefly describe what you'd do next.

Question #2

This question will analyze data on 111 different types of cars published in *Consumer Reports*. The overall goal of the analysis is to identify factors associated with price. A few key variables include:

- **Price** - List price (US dollars) with standard equipment
- **Country** - Where the car was manufactured
- **HP** - Net horsepower
- **Type** - A categorical variable describing the general type of vehicle (small, medium, large, compact, sporty, van)
- **Length** - Length of the vehicle (inches)

Part A: The plot below shows the relationship between `Type` and `Price`. Based upon the plot, do you believe these variables are associated? What is the name of the statistical test you'd use to determine whether or not there is evidence of an association?



Part B: The table below summarizes price by vehicle type. Is any information presented in this table problematic for the validity of the statistical test you identified in Part A? If so, briefly explain what aspect(s) of these data are problematic.

Type	N	Mean	Median	StdDev
Compact	19	14395.37	11650.0	5938.76
Large	7	21499.71	20225.0	5825.88
Medium	26	22750.15	23170.0	8416.81
Small	22	7736.59	7239.5	1627.93
Sporty	21	15889.81	12279.0	8539.24
Van	10	14014.30	14037.5	1126.10

Part C: The table below displays the *coefficient estimates* of a linear regression model that uses *both* Type and HP to predict a vehicle's Price. Use the information in this table to answer the following questions (I - III)

	Coefficient	Std. Error	t statistic	p-value
(Intercept)	-1842.01	2197.32	-0.84	0.40
HP	128.23	14.91	8.60	0.00
TypeLarge	2825.67	2224.20	1.27	0.21

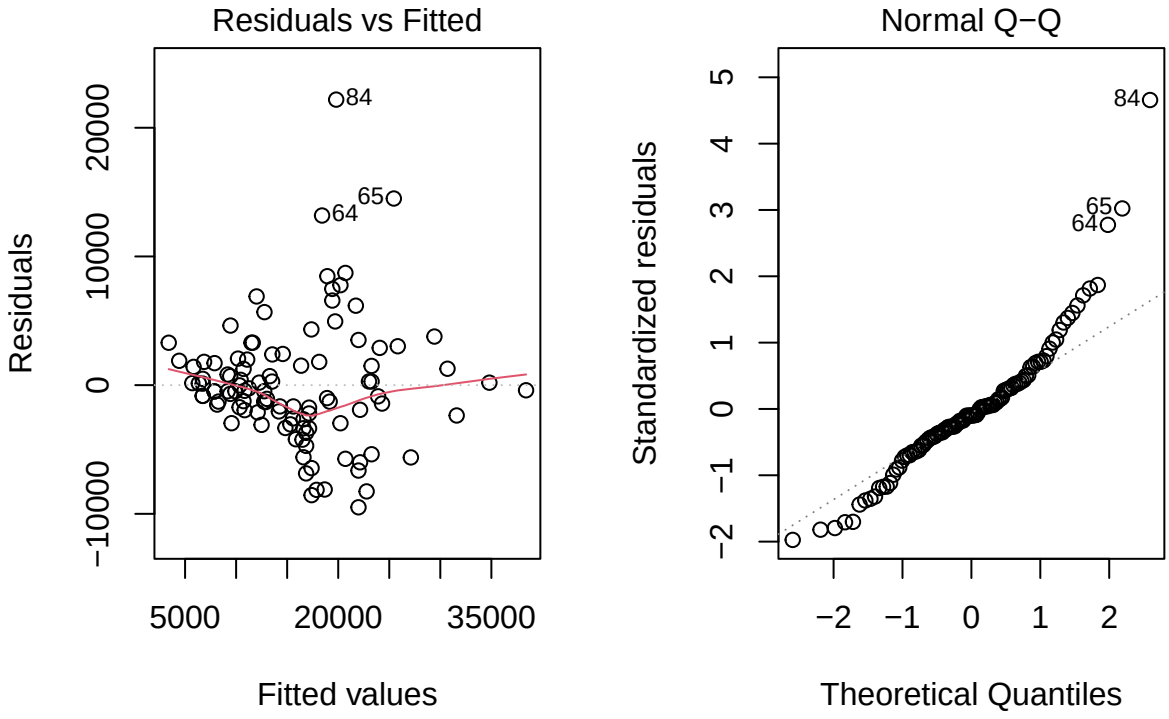
	Coefficient	Std. Error	t statistic	p-value
TypeMedium	4593.94	1543.07	2.98	0.00
TypeSmall	-1810.14	1635.77	-1.11	0.27
TypeSporty	488.56	1556.85	0.31	0.75
TypeVan	-248.79	1915.62	-0.13	0.90

I) The intercept of this model is -1842.01, what does this value mean? Should we care that this value isn't statistically significant?

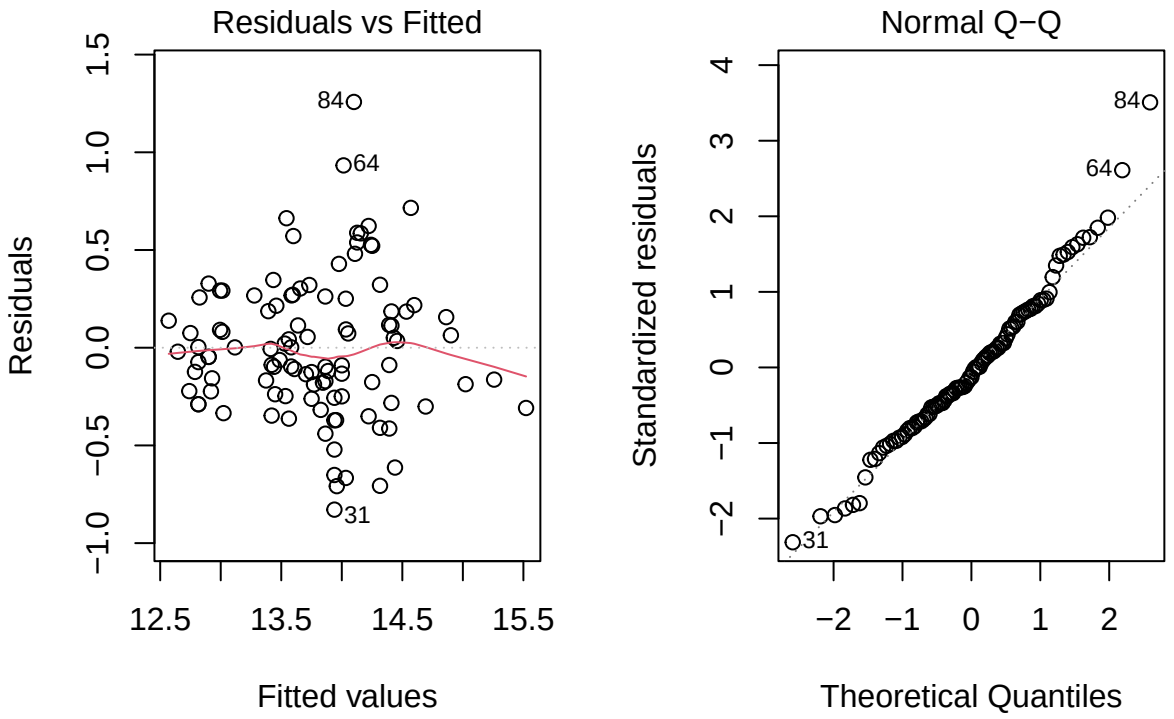
II) Provide a one sentence interpretation of the coefficient for "TypeMedium", be specific.

III) True or False, in this model the effect of HP on price differs depending on the type of vehicle. You do not need to explain your answer.

Part D: Below are two `R` plots related to the model described in Part D, `Price ~ HP + Type`. Based upon what you see in these plots, do you believe p-values calculated for these data will be valid/reliable? Briefly explain.



Part E: The plots show results after transforming the response variable `Price` using a log-transformation, making the model: `log2(Price) ~ HP + Type`. When compared with the model from Parts D-E, are you *more comfortable* trusting the p-values produced by statistical tests that involve this model? Briefly explain why or why not.



Part F: Below are statistical results found using `R`. Based upon what is given, state the null hypothesis of the test that was performed in words and provide a one-sentence conclusion describing the results of the test in regard to the null hypothesis.

```
mod0 <- lm(log2(Price) ~ HP, data = car90)
mod1 <- lm(log2(Price) ~ HP + Type, data = car90)
anova(mod0, mod1)
```

```
## Analysis of Variance Table
##
## Model 1: log2(Price) ~ HP
## Model 2: log2(Price) ~ HP + Type
##   Res.Df    RSS Df Sum of Sq      F    Pr(>F)
## 1    103 19.333
## 2     98 13.347  5    5.9861 8.7906 6.427e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Part G: Below are the estimated coefficients in `mod1` described in Part F. Interpret the coefficient of the variable `HP` in this model. Be careful to recognize that the outcome in the model has been log-transformed.

```
##  (Intercept)           HP    TypeLarge   TypeMedium    TypeSmall   TypeSporty
## 12.525579939  0.009424088  0.311799577  0.376645754 -0.473314559  0.019619029
##      TypeVan
##  0.060987375
```

Question #3

The Donner part was a famous expedition of 45 pioneers traveling to California through the Sierra Nevada mountains. The group became stranded in the mountains and spent the winter of 1846-1847 snowbound. Nearly one-half of the party starved to death before they were able to successfully escape the mountains.

Part A: Consider a Chi-squared test of independence involving the variables `sex` and `survival` using the table provided below. How many females would be expected to have died if `sex` and `survival` were independent?

```
##
##          died survived
##   Female   10       25
##   Male     32       24
```

Part B: Below are the results of the Chi-squared test mentioned in Part A. Briefly interpret these results in the context of the study and the test's hypotheses.

```
##
##  Pearson's Chi-squared test with Yates' continuity correction
##
## data:  table(Donner$sex, Donner$y)
## X-squared = 5.9719, df = 1, p-value = 0.01454
```

Part C: Below is a logistic regression model using *both* `sex` and `age` to predict survival. Interpret the estimated intercept of this model.

```
##
## Call:  glm(formula = y_binary ~ age + sex, family = "binomial", data = Donner)
##
## Coefficients:
## (Intercept)          age      sexMale
##     1.62180     -0.03561     -1.06798
##
## Degrees of Freedom: 87 Total (i.e. Null);  85 Residual
##   (3 observations deleted due to missingness)
## Null Deviance:       120.9
## Residual Deviance: 108.9     AIC: 114.9
```

Part D: Interpret the estimated coefficient of `age` in the model from Part C.

Part E: Interpret the estimated coefficient of `sexMale` in the model from Part C.

Part F: Below is a summary table for the model described in Part C. Based upon these results, do you believe that `sex` and `survival` are independent *after adjusting for differences in age*? Briefly explain.

```
##                Estimate Std. Error   z value    Pr(>|z|)
## (Intercept)  1.62180162 0.50279192  3.225592 0.001257124
## age         -0.03560529 0.01524512 -2.335521 0.019516239
## sexMale     -1.06797669 0.48228705 -2.214401 0.026801239
```